conductors to carry a larger current. Continuity and smoothness of the field must be preserved by careful joining of current inlets and outlets to avoid disturbing effects at these points. There is also the interesting possibility of so arranging the conductor system that it will form the dee system itself. A possible system having only the disadvantage that the particle must travel a short linear path each half turn on crossing the dee gap is under consideration.

A further interesting possibility related to the above is a modification of a field-producing scheme first described by Rabi[4] in which a uniform transverse magnetic field is produced in a cylindrical cavity, within and parallel to, but not concentric with, a larger conductor of circular cross section. While considered by Rabi as a long straight conductor, it may obviously be applied to the case of an annular torus of fairly large diameter. A departure from circular section in the cavity (or of the main conductor) as shown in (*c*) is required for field law and shaping. By use of such conductors it is possible that the conventional magnetic yoke of the betatron may be partly or completely dispensed with. These conductors may also be regarded as electromagnetic wave guides which may be exicted in various ways for particle acceleration.

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On the Theory of the Electrostatic Beta-Particle Energy-Spectrograph. IV

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CALCULATIONS in an earlier paper[1] of the same general title developed the basic differential equation,

$$\left.\begin{aligned} d^2\delta/d\phi^2+R(d\delta/d\phi)^2+Q\delta-qr_0=0,\\ 0\leqslant R=-C/4r_0\ll 1,\end{aligned}\right\} \tag{1}$$

for the focusing action of the electrostatic analyzer (concentric-cylinder form) on particles moving with relativistically significant speeds. In that paper the $R\delta'^2$-term was merely neglected so that a "first-order" theory could be worked out. Since this first-order theory indicated an appreciable ("chromatic") dependence of the back-focal length of the system (considered as a "particle-lens") upon particle energy, it is of interest to have explicit information bearing upon the justification for neglecting the $R\delta'^2$-term.

Although the same general result appears if Eq. (1) be studied either by approximate "equivalent linearization," or by series methods, or by obtaining from Eq. (1) an approximate "correction" to the previous solution,[1] perhaps the most satisfactory demonstration can be had by treating Eq. (1) as a "quasi-linear" differential equation. If one seeks a solution in the form (here $x=Q^{\frac{1}{2}}\phi$)

$$\delta=qr_0/Q+A_1(\phi)\sin x+A_2(\phi)\cos x, \tag{2a}$$

yet requires that

$$d\delta/d\phi=A_1(\phi)Q^{\frac{1}{2}}\cos x-A_2(\phi)Q^{\frac{1}{2}}\sin x, \tag{2b}$$

then clearly $A_1(\phi)$ and $A_2(\phi)$ must satisfy

$$\left.\begin{aligned} dA_1/d\phi=-RQ^{\frac{1}{2}}(A_1\cos x-A_2\sin x)^2\cos x,\\ dA_2/d\phi=RQ^{\frac{1}{2}}(A_1\cos x-A_2\sin x)^2\sin x;\end{aligned}\right\} \tag{3}$$

Eqs. (3) do not appear to be readily soluble in exact, closed form. But the functions,

$$\left.\begin{aligned} A_1(\phi)&=r_0\delta_i/Q^{\frac{1}{2}}l'\\ &\quad-R\delta_i^2[(H+J)\sin x-\tfrac{2}{3}I\cos^3 x-\tfrac{2}{3}J\sin^3 x],\\ A_2(\phi)&=(\delta_i-qr_0/Q)\\ &\quad-R\delta_i^2[(H-J)\cos x-\tfrac{2}{3}I\sin^3 x+\tfrac{2}{3}J\cos^3 x],\end{aligned}\right\} \tag{4a}$$

where

$$\left.\begin{aligned} H&=\tfrac{1}{2}(r_0^2/Ql'^2-1),\\ I&=-r_0/Q^{\frac{1}{2}}l',\\ J&=\tfrac{1}{2}(r_0^2/Ql'^2+1),\end{aligned}\right\} \tag{4b}$$

appear to afford a very good approximate solution to Eqs. (3); this can be seen directly by substituting Eqs. (4) into Eqs. (3) and noting that $R\delta_i$ is negligible in comparison with unity in a first-order theory.

Clearly the effects of the $R\delta'^2$-term in Eq. (1) appear in Eqs. (2) and (4), essentially as terms in the second order of δ_i which are further multiplied by the small quantity R. To this extent, therefore, that term is properly negligible in a first-order theory.

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The Production of High Centrifugal Fields

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THE maximum stress in a spinning homogeneous metal rotor is approximately proportional to the square of the peripheral speed. That is, the maximum stress the rotor will stand without bursting is proportional to $(2\pi N)^2R^2$, where N is the number of revolutions per sec. and R is the radius of the rotor. On the other hand the centrifugal acceleration at the periphery of the rotor is $(2\pi N)^2R$. Therefore, in order to produce high centrifugal fields the rotor should have a small radius R and a high rotational speed N.

A method of spinning rotors in a high vacuum previously developed in this laboratory[1] has been improved and used to spin small steel rotors up to their bursting speeds. The rotor speeds were measured by a calibrated photoelectric pick-up. The highest peripheral speed so far attained was with a 3.97-mm steel, ball-shaped rotor which burst at a peripheral speed of 9.6×10^4 cm/sec. The smallest rotor so far used was a 1.59-mm ball-shaped rotor (ball bearing with a small flat ground on top). It was spun at 166,000 r.p.s. without bursting which produced a centrifugal field of 8.8×10^7 times gravity on its periphery. With still smaller rotors it is hoped to produce correspondingly higher fields.

[1] See MacHattie, Rev. Sci. Inst. **12**, 429 (1941).